Die Aufgaben sind themenweise geordnet. Bei den mit einer Punktzahl versehenen Aufgaben handelt es sich um alte Prüfungsaufgaben zu Analysis 2 für Maschinentechnik aus den Jahren 2006 bis 2009.

Es handelt sich um viele und zum Teil sehr ähnliche Aufgaben und es ist daher nicht die Meinung, dass diese alle Aufgaben gelöst werden. Bei einem Teil der Aufgaben mag es auch genügen, wenn man sich den Lösungsweg überlegt und diesen an Hand der Lösungen überprüft.

Integrale

1. Berechne $\int \frac{x^3}{(x^2-1)(x-1)}\,dx$ 7 P

2. Berechne $\int \frac{5x+6}{x^2+3x+4}\,dx$ 7 P

3. Berechne $\int \frac{-x^2+2x-2}{(x^2+4)(x+1)}\,dx$ 10 P

4. Berechne $\int_0^{2\pi} t^2\cos(2t)\,dt$ 10 P

5. Berechne $\int \frac{x}{\sqrt{-x^2-4x-3}}\,dx$ 10 P

6. Berechne $\int \frac{2x^2}{x^3-x^2+x-1}\,dx$ 12 P

7. Berechne $\int \frac{x-2}{\sqrt{x^2-6x+5}}\,dx$ 10 P

8. Berechne $\int_0^{\frac{\pi}{2}} x^2\sin(3x)\,dx$ 9 P

9. Berechne $\int \frac{8x+9}{x^2+3x+3}\,dx$ 10 P

10. Berechne $\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} x^2\cos(2x)\,dx$ 10 P

11. Berechne $\int \frac{x^4+x+2}{x^3+x}\,dx$ 10 P

12. Berechne $\int \sqrt{-x^2-4x-3}\cdot(-6x-11)\,dx$ 10 P

13. Berechne die folgenden Integrale:

a) $\int_0^{\pi} x \cdot \sin(x)\cos(x)\, dx$

b) $\int_0^{\frac{\pi}{4}} 2\ln\left(\cos(x)\right)\tan(x)\, dx$

c) $\int_0^{\sqrt{e^2-1}} \frac{x}{1+x^2}\, dx$

d) $\int_3^6 \frac{x+1}{x^2-2x}\, dx$

e) $\int \frac{dx}{\sqrt{x+1}^3}$

f) $\int \cosh(t) \cdot e^{\sinh(t)}\, dt$

g) $\int \frac{x-1}{x^2+3x+2}\, dx$

h) $\int 2x \cdot e^{-3x}\, dx$

i) $\int_0^{\ln(2)} \ln\left(e^x+1\right)e^x\, dx$

j) $\int e^{5x+4}\, dx$

k) $\int \left(3x^2+2\right) \cdot \sin\left(x^3+2x-4\right) dx$

l) $\int_0^{\infty} \sin(2x)e^{-x}\, dx$

1. $$\int \frac{x^3}{(x^2-1)(x-1)}\,dx = I$$

$$x^3 : (x^3 - x^2 - x + 1) = 1 + \frac{x^2+x-1}{(x-1)^2(x+1)}$$ 2

$x^2 + x - 1$

$$\frac{x^2+x-1}{(x-1)^2(x+1)} = \frac{a}{x-1} + \frac{b}{(x-1)^2} + \frac{c}{x+1} \quad \Big| \cdot \text{Nenner}$$ 1

$$x^2 + x - 1 = a(x-1)(x+1) + b(x+1) + c(x-1)^2$$

$x = 1 \Rightarrow 1 = 2b \Rightarrow \underline{b = \frac{1}{2}}$

$x = -1 \Rightarrow -1 = 4c \Rightarrow \underline{c = -\frac{1}{4}}$

$x = 0 \Rightarrow -1 = -a + \frac{1}{2} - \frac{1}{4} \Rightarrow \underline{a = \frac{5}{4}}$ 2

$$\Rightarrow \underline{\underline{I}} = \int \left[1 + \frac{5}{4} \cdot \frac{1}{x-1} + \frac{1}{2} \cdot (x-1)^{-2} - \frac{1}{4} \cdot \frac{1}{x+1} \right] dx$$

$$= \underline{\underline{x + \frac{5}{4} \ln(|x-1|) - \frac{1}{2(x-1)} - \frac{1}{4} \ln(|x+1|) + c}}$$ 2

2. $\displaystyle\int \frac{5x+6}{\underbrace{x^2+3x+4}_{D=9-16<0}}\,dx$

$$= \int \frac{1}{x^2+3x+4}\left[(2x+3)\cdot\frac{5}{2} - \frac{15}{2} + \frac{12}{2}\right]dx \qquad 2$$

$$= \frac{5}{2}\cdot \ln\left(|\underbrace{x^2+3x+4}_{>0}|\right) - \frac{3}{2}\int \frac{1}{\left(x+\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{16}{4}}\,dx \qquad 1+1$$

$$= \frac{5}{2}\ln\left(x^2+3x+4\right) - \frac{3}{2}\int \frac{1}{\left(x+\frac{3}{2}\right)^2 + \frac{7}{4}}\cdot\frac{\frac{4}{7}}{\frac{4}{7}}\,dx$$

$$= \frac{5}{2}\ln\left(x^2+3x+4\right) - \frac{6}{7}\int \frac{1}{\left(\frac{2}{\sqrt{7}}x + \frac{3}{\sqrt{7}}\right)^2 + 1}\,dx \qquad 2$$

$$= \frac{5}{2}\ln\left(x^2+3x+4\right) - \frac{3}{\sqrt{7}}\arctan\left(\frac{2x+3}{\sqrt{7}}\right) + c \qquad 1$$

3. $$\int \frac{-x^2+2x-2}{(x^2+4)(x+1)}\,dx = I$$

$$\frac{-x^2+2x-2}{(x^2+4)(x+1)} = \frac{ax+b}{x^2+4} + \frac{c}{x+1} \qquad \Big| \cdot \text{Nenner}$$ 2

$$-x^2+2x-2 = (ax+b)(x+1) + c(x^2+4)$$ 1

$$= ax^2+ax+bx+b+cx^2+4c$$

$$= x^2\underbrace{(a+c)}_{-1} + x\underbrace{(a+b)}_{2} + \underbrace{b+4c}_{-2}$$

$$x=-1 \Rightarrow -5 = 5c \Rightarrow \underline{c=-1} \Rightarrow \underline{a=0}\,,\ \underline{b=2}$$ 3

$$\Rightarrow \underline{\underline{I}} = \int\left(\frac{2}{x^2+4} - \frac{1}{x+1}\right)dx$$

$$= \int\left(\frac{2}{4}\cdot\frac{1}{\left(\frac{x}{2}\right)^2+1} - \frac{1}{x+1}\right)dx$$

$$= \frac{1}{2}\arctan\left(\frac{x}{2}\right)\cdot 2 - \ln(|x+1|) + c$$

$$= \underline{\underline{\underbrace{\arctan\left(\frac{x}{2}\right)}_{\underline{3}} - \underbrace{\ln(|x+1|)}_{\underline{1}} + c}}$$ 4

4. $$\underline{\underline{\int_0^{2\pi} \underset{u(t)}{t^2} \underset{v'(t)}{\cos(2t)}\, dt}}$$

$$= \underbrace{\left[t^2 \cdot \sin(2t) \cdot \frac{1}{2} \right]_0^{2\pi}}_{0-0} - \frac{2}{2} \int_0^{2\pi} \underset{\downarrow}{t}\ \underset{\uparrow}{\sin(2t)}\, dt$$

$\underline{2} + \underline{2}$

$$= - \left[\underbrace{\left[-t \cdot \cos(2t) \cdot \frac{1}{2} \right]_0^{2\pi}}_{-\pi + 0} + \frac{1}{2} \int_0^{2\pi} \cos(2t)\, dt \right]$$

$\underline{2} + \underline{2}$

$$= \pi - \frac{1}{2} \underbrace{\left[\sin(2t) \cdot \frac{1}{2} \right]_0^{2\pi}}_{0-0}$$

$$= \underline{\underline{\pi}}$$

$\underline{2}$

5. $\displaystyle\int \frac{x}{\sqrt{-x^2-4x-3}}\,dx$

$$= \int \Bigg[\underbrace{\frac{-2x-4}{\sqrt{-x^2-4x-3}}}_{f(x)^{-\frac{1}{2}} \cdot f'(x)} \cdot \frac{1}{-2} - 2 \cdot \frac{1}{\sqrt{-\left[x^2+4x+3\right]}} \Bigg]\,dx \qquad \underline{2}+\underline{2}$$

$$= -\frac{1}{2} \cdot \frac{\sqrt{-x^2-4x-3}}{\frac{1}{2}} - 2\int \frac{1}{\sqrt{-\left[(x+2)^2-1\right]}}\,dx$$

$$= -\sqrt{-x^2-4x-3} - 2\int \frac{1}{\sqrt{1-(x+2)^2}}\,dx$$

$$= \underline{\underline{-\sqrt{-x^2-4x-3} - 2\arcsin(x+2) + c}} \qquad \underline{2}+\underline{4}$$

6. $\displaystyle\int \frac{2x^2}{x^3-x^2+x-1}\,dx = I$

Eine NS des Nenners erraten : 1 1) $\underline{1}$

Horner :

$$\begin{array}{c|ccc|c} & 1 & -1 & 1 & -1 \\ x=1 & & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & 0 \end{array} \checkmark$$

$\underbrace{1 \quad 0 \quad 1}_{x^2+1}$: hat keine reelle NS $\Rightarrow$ $\underline{2}$

$$\frac{2x^2}{x^3-x^2+x-1} = \frac{2x^2}{(x-1)(x^2+1)} = \frac{a}{x-1} + \frac{bx+c}{x^2+1} \quad \Big| \cdot \text{Nenner}$$ $\underline{1}$

$$\Rightarrow 2x^2 = a(x^2+1) + (bx+c)(x-1)$$
$$= ax^2 + a + bx^2 - bx + cx - c$$
$$= x^2\underbrace{(a+b)}_{2} + x\underbrace{(-b+c)}_{0} + \underbrace{(a-c)}_{0}$$

$x = 1 \Rightarrow 2 = 2a \Rightarrow \underline{a=1} \Rightarrow \underline{b=1}\,,\ \underline{c=1}$ $\underline{3}$

$$\Rightarrow \underline{\underline{I}} = \int \left[\frac{1}{x-1} + \frac{x+1}{x^2+1}\right] dx$$

$$= \ln(|x-1|) + \int \underbrace{\frac{1}{x^2+1}}_{f(x)^{-1}} \Big[\underbrace{2x}_{f'(x)} \cdot \frac{1}{2} + 1\Big]\, dx$$ $\underline{1} + \underline{3}$

$$= \underline{\underline{\ln(|x-1|) + \frac{1}{2}\ln(x^2+1) + \arctan(x) + c}}$$ $\underline{1}$

1) Nenner kann hier auch direkt faktorisiert werden:
$x^3-x^2+x-1 = x^2(x-1)+(x-1) = (x-1)(x^2+1)$

$$7.\quad \int \frac{x-2}{\sqrt{x^2-6x+5}}\,dx$$

$$= \int \underbrace{\frac{1}{\sqrt{x^2-6x+5}}}_{f(x)^{-\frac{1}{2}}} \left[\underbrace{(2x-6)}_{f'(x)} \cdot \frac{1}{2} + (3-2) \right] dx \qquad \underline{3}$$

$$= \frac{1}{2} \cdot \sqrt{x^2-6x+5} \cdot \frac{1}{\frac{1}{2}} + \int \frac{1}{\sqrt{(x-3)^2-4}}\,dx \qquad \underline{1}+\underline{1}$$

$$= \sqrt{x^2-6x+5} + \int \frac{1}{\sqrt{4\left[\left(\frac{x-3}{2}\right)^2-1\right]}}\,dx$$

$$= \sqrt{x^2-6x+5} + \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{x-3}{2}\right)^2-1}}\,dx \qquad \underline{2}$$

$$= \sqrt{x^2-6x+5} + \frac{1}{2} \underbrace{\ln\left(\left|\frac{x-3}{2} + \sqrt{\left(\frac{x-3}{2}\right)^2-1}\right|\right)}_{\ln\left(\frac{\left|x-3+\sqrt{(x-3)^2-4}\right|}{2}\right)} \cdot 2 + c_1 \qquad \underline{3}$$

$$= \underline{\underline{\sqrt{x^2-6x+5} + \ln\left(\left|x-3+\sqrt{x^2-6x+5}\right|\right) + c}}$$

8. $$\int_0^{\frac{\pi}{2}} \underbrace{x^2}_{u(x)} \cdot \underbrace{\sin(3x)}_{v'(x)} \, dx$$

$$= \underbrace{\left[x^2 \cdot (-1) \cos(3x) \cdot \frac{1}{3} \right]_0^{\frac{\pi}{2}}}_{0-0} + \frac{2}{3} \int_0^{\frac{\pi}{2}} \underset{\downarrow}{x} \cdot \underset{\uparrow}{\cos(3x)} \, dx \qquad \underline{1} + \underline{1}$$

$\underline{1}$

$$= \frac{2}{3} \left(\left[x \cdot \sin(3x) \cdot \frac{1}{3} \right]_0^{\frac{\pi}{2}} - \frac{1}{3} \int_0^{\frac{\pi}{2}} \sin(3x) \, dx \right) \qquad \underline{1} + \underline{1}$$

$$= \frac{2}{9} \left(\frac{\pi}{2} \cdot (-1) - 0 \right) - \frac{2}{9} \underbrace{\left[-\cos(3x) \cdot \frac{1}{3} \right]_0^{\frac{\pi}{2}}}_{0 + \frac{1}{3}} \qquad \underline{1} + \underline{1}$$

$\underline{1}$

$$= \underline{\underline{-\frac{\pi}{9} - \frac{2}{27}}} \qquad \underline{1}$$

9. $$\int \frac{8x+9}{\underbrace{x^2+3x+3}_{D\,=\,9-12\,<\,0}}\,dx$$

$$= \int \underbrace{\frac{1}{x^2+3x+3}}_{\frac{1}{f(x)}} \Big[\underbrace{(2x+3)}_{f'(x)}\cdot 4 \;-12+9\Big]\,dx \qquad \underline{3}$$

$$= 4\ln\big(\underbrace{|x^2+3x+3|}_{>0}\big) - 3\int \frac{1}{\left(x+\frac{3}{2}\right)^2-\frac{9}{4}+\frac{12}{4}}\,dx$$

$$= 4\ln\left(x^2+3x+3\right) - 3\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\frac{3}{4}}\cdot\frac{\frac{4}{3}}{\frac{4}{3}}\,dx \qquad \underline{2}+\underline{1}$$

$$= 4\ln\left(x^2+3x+3\right) - 4\int \frac{1}{\left(\frac{2}{\sqrt{3}}\right)^2\left(x+\frac{3}{2}\right)^2+1}\,dx$$

$$= 4\ln\left(x^2+3x+3\right) - 4\int \frac{1}{\left(\frac{2}{\sqrt{3}}x+\frac{3}{\sqrt{3}}\right)^2+1}\,dx \qquad \underline{2}$$

$$= 4\ln\left(x^2+3x+3\right) - 4\arctan\left(\frac{2x+3}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{2} + c$$

$$= \underline{\underline{4\ln\left(x^2+3x+3\right) - 2\sqrt{3}\cdot\arctan\left(\frac{2x+3}{\sqrt{3}}\right) + c}} \qquad \underline{2}$$

10. $$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \underbrace{x^2}_{u(x)} \cdot \underbrace{\cos(2x)}_{v'(x)}\, dx$$

$$= \left[x^2 \cdot \frac{\sin(2x)}{2}\right]_{-\frac{\pi}{4}}^{\frac{\pi}{2}} - \frac{2}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \underset{\downarrow}{x} \cdot \underset{\uparrow}{\sin(2x)}\, dx$$ $\underline{1}+\underline{1}$

$$= 0 - \frac{\pi^2}{16} \cdot \left(\frac{-1}{2}\right) - \left(\underbrace{\left[x \cdot \frac{-\cos(2x)}{2}\right]_{-\frac{\pi}{4}}^{\frac{\pi}{2}}}_{\frac{\pi}{2} \cdot \frac{1}{2} - \left(-\frac{\pi}{4}\right) \cdot 0} + \frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(2x)\, dx \right)$$ $\underline{2}+\underline{1}+\underline{1}$

$\underline{2}$

$$= \frac{\pi^2}{32} - \frac{\pi}{4} - \frac{1}{2} \cdot \frac{1}{2} \underbrace{\left[\sin(2x)\right]_{-\frac{\pi}{4}}^{\frac{\pi}{2}}}_{0-(-1)}$$ $\underline{1}$

$$= \underline{\underline{\frac{\pi^2}{32} - \frac{\pi}{4} - \frac{1}{4}}}$$ $\underline{1}$

11. $\displaystyle\int \frac{x^4+x+2}{x^3+x}\,dx = I$

$$(x^4+x+2):(x^3+x) = x + \frac{-x^2+x+2}{x(x^2+1)}$$ 2

$$\frac{\begin{array}{l}(x^4+x+2) \\ -(x^4+x^2)\end{array}}{\qquad -x^2+x+2}$$

$$\frac{-x^2+x+2}{x(x^2+1)} = \frac{a}{x} + \frac{bx+c}{x^2+1}$$ 1

$$\Rightarrow -x^2+x+2 = a(x^2+1) + (bx+c)\cdot x$$
$$= x^2\underbrace{(a+b)}_{-1} + \underbrace{c}_{1}x + \underbrace{a}_{2}$$

$$\Rightarrow \underline{a=2}, \quad \underline{b=-3}, \quad \underline{c=1}$$ 3

$$\Rightarrow \underline{\underline{I}} = \int\left[x + \frac{2}{x} + \frac{-3x+1}{x^2+1}\right]dx$$

$$= \frac{x^2}{2} + 2\ln(|x|) + \int \underbrace{\frac{1}{x^2+1}}_{f(x)^{-1}}\left[\underbrace{2x}_{f'(x)}\cdot\frac{-3}{2} + 1\right]dx$$

$$= \underline{\underline{\frac{x^2}{2} + 2\ln(|x|) - \frac{3}{2}\ln(x^2+1) + \arctan(x) + c}}$$ 4

$\underline{1}$ $\quad\underline{1}\quad$ $\underline{1}$ $\quad\underline{1}$

12. $\int \sqrt{-x^2-4x-3}\cdot(-6x-11)\,dx$

$$= \int \underbrace{\sqrt{-x^2-4x-3}}_{f(x)^{\frac{1}{2}}}\left[\underbrace{(-2x-4)}_{f'(x)}\cdot 3 + 1\right]dx \qquad 2$$

$$= 3\,\frac{(-x^2-4x-3)^{\frac{3}{2}}}{\frac{3}{2}} + \int\sqrt{-\left[x^2+4x+3\right]}\,dx \qquad 2+1$$

$$= 2\sqrt{-x^2-4x-3}^{\,3} + \int\sqrt{-\left[(x+2)^2-1\right]}\,dx \qquad 2$$

$$= 2\sqrt{-x^2-4x-3}^{\,3} + \int\sqrt{1-(x+2)^2}\,dx \qquad 1$$

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$$= \underline{\underline{2\sqrt{-x^2-4x-3}^{\,3} + \frac{1}{2}\left[(x+2)\cdot\sqrt{1-(x+2)^2} + \arcsin(x+2)\right] + c}} \qquad 2$$

13. a) $$\int_0^{\pi} x \cdot \sin(x)\cos(x)\,dx = \frac{1}{2}\int_0^{\pi} \underbrace{x}_{u(x)} \cdot \underbrace{\sin(2x)}_{v'(x)}\,dx$$

$$= \frac{1}{2}\left[\left[\underbrace{x}_{u(x)} \cdot \underbrace{(-1)\,\frac{\cos(2x)}{2}}_{v(x)}\right]_0^{\pi} + \frac{1}{2}\int_0^{\pi} \cos(2x)\,dx\right]$$

$$= \frac{1}{2}\left[-\frac{\pi}{2} - 0 + \underbrace{\frac{1}{4}\left[\sin(2x)\right]_0^{\pi}}_{0-0}\right] = \underline{\underline{-\frac{\pi}{4}}}$$

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b) $$\int_0^{\frac{\pi}{4}} 2\ln(\cos(x))\tan(x)\,dx$$

$$= -2\int_0^{\frac{\pi}{4}} \underbrace{\ln(\cos(x))}_{f(x)^1} \cdot \underbrace{\frac{1}{\cos(x)} \cdot (-1)\sin(x)}_{f'(x)}\,dx$$

$$= -2\left[\frac{\ln(\cos(x))^2}{2}\right]_0^{\frac{\pi}{4}} = -\left[\ln\left(\cos\left(\frac{\pi}{4}\right)\right)^2 - \ln(\cos(0))^2\right]$$

$$= -\left[\ln\left(\frac{1}{\sqrt{2}}\right)^2 - \underbrace{\ln(1)^2}_{0}\right] = -\left[\left(\underbrace{\ln(1)}_{0} - \ln(\sqrt{2})\right)^2\right]$$

$$= \underline{\underline{-\ln^2(\sqrt{2})}} = -\ln^2\left(2^{\frac{1}{2}}\right) = -\left[\frac{1}{2}\ln(2)\right]^2$$

$$= \underline{\underline{-\frac{1}{4}\ln^2(2)}}$$

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c) $$\int_0^{\sqrt{e^2-1}} \frac{x}{1+x^2}\,dx = \frac{1}{2}\int_0^{\sqrt{e^2-1}} \underbrace{\frac{2x}{1+x^2}}_{\frac{f'(x)}{f(x)}}\,dx = \frac{1}{2}\left[\ln\left(\left|1+x^2\right|\right)\right]_0^{\sqrt{e^2-1}}$$

$$= \frac{1}{2}\left[\ln(e^2) - \underbrace{\ln(1)}_{0}\right] = \underline{\underline{1}}$$

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13. d) $\displaystyle\int_3^6 \frac{x+1}{x^2-2x}\,dx = I$

$$\frac{x+1}{x(x-2)} = \frac{a}{x} + \frac{b}{x-2} \quad \Big| \cdot x(x-2)$$

$$x+1 = a(x-2) + bx$$

$$x = 0 \Rightarrow 1 = -2a \Rightarrow \underline{a = -\tfrac{1}{2}}$$

$$x = 2 \Rightarrow 3 = 2b \Rightarrow \underline{b = \tfrac{3}{2}}$$

$$\Rightarrow I = \int_3^6 \left[-\frac{1}{2}\cdot\frac{1}{x} + \frac{3}{2}\cdot\frac{1}{x-2}\right] dx$$

$$= -\frac{1}{2}\Big[\ln(|x|)\Big]_3^6 + \frac{3}{2}\Big[\ln(|x-2|)\Big]_3^6$$

$$= -\frac{1}{2}\underbrace{\Big[\ln(6) - \ln(3)\Big]}_{\ln\left(\frac{6}{3}\right)} + \frac{3}{2}\Big[\underbrace{\ln(4)}_{\ln(2^2)} - \underbrace{\ln(1)}_{0}\Big]$$

$$= -\frac{1}{2}\ln(2) + \frac{3}{2}\cdot 2\ln(2) = \underline{\underline{\frac{5}{2}\ln(2)}}$$

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e) $\displaystyle\int \frac{dx}{\sqrt{x+1}^{\,3}} = \int (x+1)^{-\frac{3}{2}}\,dx = \frac{(x+1)^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$$= \underline{\underline{-\frac{2}{\sqrt{x+1}} + c}}$$

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f) $\displaystyle\int \underbrace{\cosh(t)}_{g'(t)} \cdot \underbrace{e^{\sinh(t)}}_{f(g(t))}\,dt = \int e^x\,dx = e^x + c$

$x = \sinh(t)$, $dx = \cosh(t)\,dt$

$$= \underline{\underline{e^{\sinh(t)} + c}}$$

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13 g) $\displaystyle\int \frac{x-1}{x^2+3x+2}\,dx = I$

$$\frac{x-1}{(x+2)(x+1)} = \frac{a}{x+2} + \frac{b}{x+1} \quad \Big|\cdot (x+2)(x+1)$$

$$x-1 = a(x+1) + b(x+2)$$

$x=-1 \Rightarrow \underline{-2 = b}$
$x=-2 \Rightarrow -3 = -a \Rightarrow \underline{a=3}$

$$\Rightarrow I = \int\left[\frac{3}{x+2} - \frac{2}{x+1}\right]dx = \underline{\underline{3\ln(|x+2|) - 2\ln(|x+1|) + c}}$$

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h) $\displaystyle\int \underset{\downarrow}{2x}\cdot \underset{\uparrow}{e^{-3x}}\,dx = 2x\cdot e^{-3x}\cdot\frac{1}{-3} - \frac{2}{-3}\int e^{-3x}\,dx$

$$= -\frac{2}{3}x e^{-3x} - \frac{2}{9}e^{-3x} + c$$

$$= \underline{\underline{-\frac{2}{9}e^{-3x}(3x+1) + c}}$$

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i) $\displaystyle\int_0^{\ln(2)} \ln(\underbrace{e^x+1}_{g(x)})\underbrace{e^x}_{g'(x)}\,dx = \int_2^3 \ln(u)\,du = \int_2^3 \underset{\uparrow}{1}\cdot \underset{\downarrow}{\ln(u)}\,du$

$u = e^x+1, \quad du = e^x dx$

$$= \Big[u\cdot\ln(u)\Big]_2^3 - \int_2^3 u\cdot\frac{1}{u}\,du$$

$$= 3\ln(3) - 2\ln(2) - \Big[u\Big]_2^3$$

$$= \underline{\underline{\ln\left(\frac{27}{4}\right) - 1}}$$

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13 j) $\int e^{5x+4}\,dx = \underline{\underline{e^{5x+4}\cdot\frac{1}{5}+c}}$

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k) $\int \underbrace{(3x^2+2)}_{g'(x)} \cdot \sin(\underbrace{x^3+2x-4}_{g(x)})\,dx$

$= \int \sin(t)\,dt = -\cos(t)+c$

$t = g(x)\,,\quad dt = g'(x)\,dx$

$= \underline{\underline{-\cos(x^3+2x-4)+c}}$

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l) $\int_0^\infty \sin(2x)\cdot e^{-x}\,dx = \left[\frac{e^{-x}}{5}\left[-\sin(2x)-2\cos(2x)\right]\right]_0^\infty$

Integraltabelle Nr. 26 : $a=-1$; $b=2$

$= 0-\frac{1}{5}\left[-0-2\right] = \underline{\underline{\frac{2}{5}}}$

$e^{-x} \xrightarrow{x\to\infty} 0$

$-\sin(2x)-2\cos(2x)$ hat höchstens den Betrag 3
