

# Musterlösung Faltung

$$p(t) = \begin{cases} t & ; 0 \leq t < 1 \\ 2-t & ; 1 \leq t < 3 \\ 0 & ; \text{sonst} \end{cases}$$

$$g(t) = \begin{cases} 1 & ; -1 \leq t < 1 \\ 0 & ; \text{sonst} \end{cases}$$

$$p(u) = \begin{cases} u & ; 0 \leq u < 1 \\ 2-u & ; 1 \leq u < 3 \\ 0 & ; \text{sonst} \end{cases}$$

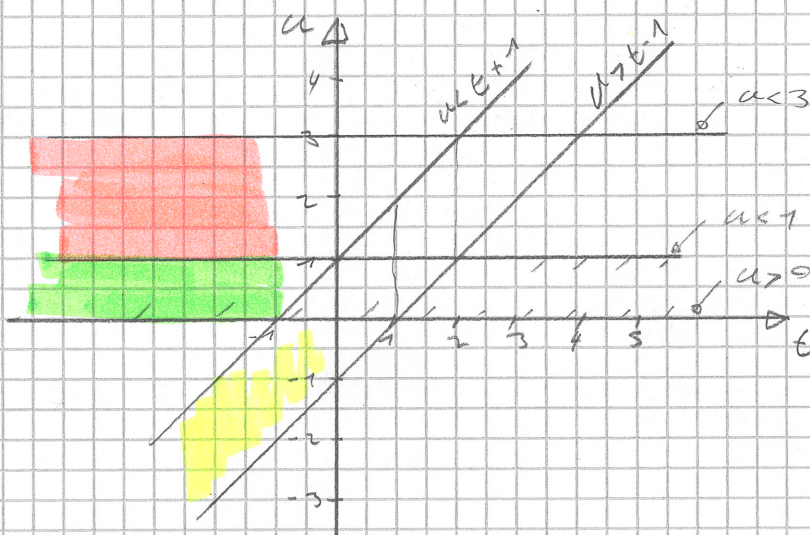
$$\rightarrow u > 0 ; u < 1$$

$$\rightarrow u > 1 ; u < 3$$

$$g(t-u) = \begin{cases} 1 & ; -1 \leq t-u < 1 \\ 0 & ; \text{sonst} \end{cases}$$

$$\begin{aligned} -1 \leq t-u < 1 & \Rightarrow -1 \leq t-u \\ -1 \leq t-u & \Rightarrow -1-t \leq -u \\ -u > -1-t & \Rightarrow u < t+1 \end{aligned}$$

$$\begin{aligned} t-u < 1 & \Rightarrow u < t-1 \\ u > t-1 & \Rightarrow u > t-1 \end{aligned}$$



$$-1 \leq t < 0 \quad \int_0^{t+1} u \, du = \left[ \frac{u^2}{2} \right]_0^{t+1}$$

$$0 \leq t < 1 \quad \int_0^1 u \, du + \int_1^{t+1} (2-u) \, du = \left[ \frac{u^2}{2} \right]_0^1 + \left[ 2u - \frac{u^2}{2} \right]_1^{t+1}$$

$$1 \leq t < 2 \quad \int_{t-1}^1 u \, du + \int_1^{t+1} (2-u) \, du = \left[ \frac{u^2}{2} \right]_{t-1}^1 + \left[ 2u - \frac{u^2}{2} \right]_1^{t+1}$$

$$2 \leq t < 4 \quad \int_{t-1}^3 (2-u) \, du = \left[ 2u - \frac{u^2}{2} \right]_{t-1}^3$$

$$-1 \leq t < 0 \quad \frac{(t+1)^2}{2} = \frac{1}{2} (t^2 + 2t + 1)$$

$$0 \leq t < 1 \quad \frac{1}{2} + 2(t+1) - \frac{(t+1)^2}{2} - 2 + \frac{1}{2} = \frac{t^2}{2} + t + \frac{1}{2}$$

$$1 \leq t < 2 \quad \frac{1}{2} - \frac{(t-1)^2}{2} + 2(t+1) - \frac{(t+1)^2}{2} - 2 + \frac{1}{2} = -\frac{t^2}{2} + 2t$$

$$2 \leq t < 4 \quad 6 - \frac{9}{2} - 2t + 2 + \frac{(t-1)^2}{2} = \frac{t^2}{2} - 3t + 4$$

# Musterlösung Fouriertransformierte + Phasendichte

$$s(t) = \begin{cases} \cos^2(2t) & ; -\pi < t < \pi \\ 0 & ; \text{sonst} \end{cases} \quad \left| \begin{array}{l} \text{Phasendichte im Bereich} \\ -4 \dots 4 \text{ gesucht} \end{array} \right.$$

$$\Rightarrow s(t) = \begin{cases} \frac{1}{2} (1 + \cos(4t)) & ; -\pi < t < \pi \\ 0 & ; \text{sonst} \end{cases}$$

$$S(\omega) = \int_{-\infty}^{\infty} s(t) \cdot e^{-j\omega t} dt$$

$$\Rightarrow \frac{1}{2} \cdot \int_{-\pi}^{\pi} (1 + \cos(4t)) \cdot e^{-j\omega t} dt$$

$$= \frac{1}{2} \cdot \int_{-\pi}^{\pi} e^{-j\omega t} dt + \frac{1}{2} \cdot \int_{-\pi}^{\pi} \frac{1}{2} \cdot (e^{+4jt} + e^{-4jt}) \cdot e^{-j\omega t} dt$$

$$= \frac{1}{2} \cdot \left[ \frac{e^{-j\omega t}}{-j\omega} \right]_{-\pi}^{\pi} + \frac{1}{4} \cdot \int_{-\pi}^{\pi} e^{j\omega(4-t)} + e^{-j\omega(4+t)} dt$$

$$= \frac{1}{2} \cdot \left[ \frac{e^{-j\omega\pi}}{-j\omega} - \frac{e^{j\omega\pi}}{-j\omega} \right] + \frac{1}{4} \cdot \left[ \frac{e^{j\omega(4-t)}}{j(4-\omega)} + \frac{e^{-j\omega(4+t)}}{-j(4+\omega)} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\omega} \cdot \frac{1}{2j} \cdot (e^{-j\omega\pi} - e^{j\omega\pi}) + \frac{1}{4j(4-\omega)} \cdot (e^{j\omega(4-\pi)} - e^{-j\omega(4-\pi)}) + \frac{1}{4j(4+\omega)} \cdot (e^{-j\omega(4+\pi)} - e^{j\omega(4+\pi)})$$

$$= \frac{1}{\omega} \cdot \sin(\omega\pi) + \frac{1}{2(4-\omega)} \cdot \sin(\pi(4-\omega)) + \frac{1}{2(4+\omega)} \cdot \sin(\pi(4+\omega))$$

$$= \frac{1}{\omega} \cdot \sin(\omega\pi) + \frac{1}{8-2\omega} \cdot \sin(4\pi - \pi\omega) + \frac{1}{8+2\omega} \cdot \sin(4\pi + \pi\omega)$$

$$= \frac{1}{\omega} \cdot \sin(\pi\omega) - \frac{1}{8-2\omega} \cdot \sin(\pi\omega) + \frac{1}{8+2\omega} \cdot \sin(\pi\omega)$$

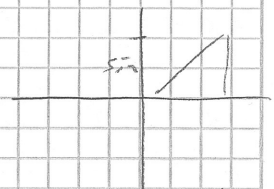
$$\sin(\pi\omega) \cdot \left( \frac{1}{\omega} - \frac{1}{8-2\omega} + \frac{1}{8+2\omega} \right) =$$

$$= \sin(\pi\omega) \cdot \left( \frac{(8-2\omega)(8+2\omega) - \omega(8+2\omega) + \omega(8-2\omega)}{\omega \cdot (8-2\omega)(8+2\omega)} \right)$$

$$= \sin(\pi\omega) \cdot \left( \frac{64 - 8\omega^2}{4\omega(4-\omega)(4+\omega)} \right)$$

Am Ende muss ein Produkt stehen.

Nur von 0...π auswerfen = Symmetrie.



## Musterlösung Anfangswertproblem

$$y'''(t) + 4y'(t) = \sigma(t) ; y(0) = -1 ; y'(0) = 0 ; y'''(0) = 2$$

$$\sigma(t) \xrightarrow{\quad} \frac{1}{s}$$

$$y(t) \xrightarrow{\quad} y(s)$$

$$y'(t) \xrightarrow{\quad} s \cdot y(s) - (-1)$$

$$y''(t) \xrightarrow{\quad} s^2 \cdot y(s) + s \cdot 1 - 0$$

$$y'''(t) \xrightarrow{\quad} s^3 \cdot y(s) + s^2 - 2$$

$$\rightarrow s^3 y(s) + s^2 - 2 + 4s \cdot y(s) + 4 = \frac{1}{s}$$

$$y(s)(s^3 + 4s) = -s^2 + \frac{1}{s} - 2$$

$$y(s) = \frac{-s^2 + \frac{1}{s} - 2}{s^3 + 4s}$$

$$y(s) = \frac{-s^3 - 2s + 1}{s^2(s^2 + 4)} \rightarrow \text{partialbruch}$$

$$\frac{a}{s} + \frac{b}{s^2} + \frac{cs+d}{s^2+4} = \frac{-s^3 - 2s + 1}{s^2(s^2+4)}$$

$$as(s^2+4) + b(s^2+4) + cs^3 + ds^2 = -s^3 - 2s + 1$$

$$(a+c)s^3 = -1 \rightarrow -\frac{1}{2} + c = -1 \Rightarrow c = -\frac{1}{2}$$

$$(b+d)s^2 = 0 \rightarrow \frac{1}{4} + d = 0 \Rightarrow d = -\frac{1}{4}$$

$$4as = -2 \rightarrow a = -\frac{2}{4} = -\frac{1}{2}$$

$$4b = 1 \rightarrow b = \frac{1}{4}$$

$$y(s) = -\frac{1}{2} \cdot \frac{1}{s} + \frac{1}{4} \cdot \frac{1}{s^2} - \frac{1}{2} \cdot \frac{s + \frac{1}{2}}{s^2 + 4}$$

$$y(s) = -\frac{1}{2} \cdot \frac{1}{s} + \frac{1}{4} \cdot \frac{1}{s^2} - \frac{1}{2} \cdot \frac{s}{s^2+4} - \frac{1}{8} \cdot \frac{2}{s^2+4}$$

$$y(t) = -\frac{1}{2} + \frac{1}{4}t - \frac{1}{2}\cos(2t) - \frac{1}{8}\sin(2t)$$

## Musterlösung Laplace transformation

$$f(t) = \sum_{k=0}^{\infty} p(t - k\pi) \cdot \cos(t)$$

$$= \sum_{k=0}^{\infty} p(t - k\pi) \cdot \cos(k\pi) = \sum_{k=0}^{\infty} (-1)^k \cdot p(t - k\pi)$$

$$F(s) = \sum_{k=0}^{\infty} (-1)^k \cdot e^{-k\pi s}$$

$$= \sum_{k=0}^{\infty} (-1)^k \cdot e^{(-\pi s)k} \rightarrow \text{alternierende geometrische Reihe}$$

$$\rightarrow F(s) = \frac{e^{\pi s}}{1 + e^{\pi s}}$$